Singapore–Cambridge General Certificate of Education
Advanced Level Higher 2 (2027)

Further Mathematics
(Syllabus 9649)

CONTENTS

PREAMBLE

Mathematics drives many of the advancements in sciences, engineering, economics and technology. It is at the heart of many of the innovative products and services today. A strong grounding in mathematics is essential for students who aspire to be scientists or engineers or any other professionals who require mathematical tools to solve complex problems.

H2 Further Mathematics is designed for students who are mathematically-inclined and who intend to specialise in mathematics, sciences or engineering or disciplines with higher demand on mathematical skills. It extends and expands on the range of mathematics and statistics topics in H2 Mathematics and provides these students with a head start in learning a wider range of mathematical methods and tools that are useful for solving more complex problems in mathematics and statistics.

H2 Further Mathematics is to be offered with H2 Mathematics as a double mathematics course.

SYLLABUS AIMS

The aims of H2 Further Mathematics are to enable students to:

(a) acquire a wider range of mathematical concepts and stronger set of mathematical skills for their tertiary studies in mathematics, sciences, engineering and other related disciplines with a heavier demand on mathematics

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem solving

(c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines

(d) experience and appreciate the rigour and abstraction in the discipline.

ASSESSMENT OBJECTIVES (AO)

The assessment will test candidates' abilities to:

AO1   **Use mathematical techniques and procedures**
- recall facts, formulae and notation and use them directly
- read and use information from tables, graphs, diagrams and texts
- carry out straightforward mathematical procedures

AO2   **Formulate and solve problems including those in real-world contexts**
- select relevant mathematical concept or strategy to apply
- formulate problems into mathematical expressions or models
- integrate mathematical concepts to solve mathematical problems
- translate between equivalent forms of mathematical expressions or statements
- interpret results in the context of a given problem

AO3   **Reason and communicate mathematically**
- explain the choice of mathematical models or strategies
- make deductions, inferences and generalisations
- formulate conjectures and justify mathematical statements
- construct mathematical arguments and proofs

Approximate weightings for the assessment objectives are as follows:

AO1	30%
AO2	60%
AO3	10%

USE OF A GRAPHING CALCULATOR (GC)

The use of an approved GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates must present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Candidates should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

LIST OF FORMULAE AND RESULTS

Candidates will be provided in the examination with a list of formulae and results.

INTEGRATION AND APPLICATION

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

Possible list of H2 Further Mathematics applications and contexts:

Applications and contexts	Some possible topics involved
Kinematics and dynamics (e.g. free fall, projectile motion, orbital motion, collisions)	Functions; Calculus; Vectors
Movie graphics	Vectors
Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits (including alternating current circuit)	Complex numbers; Calculus
Population growth (e.g. spread of diseases), radioactive decay, heating and cooling problems, mixing, chemical changes, charging	Differential equations
Search engines, cryptography, digital music	Matrices and linear spaces

Applications and contexts	Some possible topics involved
Financial maths (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression
Polling	Confidence intervals; Hypothesis testing
Genetics	Chi-square tests

The list illustrates some types of contexts in which the mathematics learnt in the syllabus may be applied and is by no means exhaustive. While problems may be set based on these contexts, no assumptions will be made about the knowledge of these contexts. All information will be self-contained within the problem.

SCHEME OF EXAMINATION PAPERS

For the examination in H2 Further Mathematics, there will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

PAPER 1 (3 hours)
A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be one question on the application of mathematics in real-world contexts, including those from sciences and engineering. The question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

PAPER 2 (3 hours)
A paper consisting of two sections, Sections A and B.

Section A (Pure Mathematics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Pure Mathematics section (i.e. Algebra and Calculus, and Discrete Mathematics, Matrices and Numerical Methods) of the syllabus.

Section B (Probability and Statistics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be one question in Section B on application of mathematics in real-world contexts, including those from sciences and engineering. The question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

CONTENT OUTLINE

Knowledge of the content of H2 Mathematics is assumed.

	Topic/Sub-topics	Content
SECTION A: PURE MATHEMATICS		
1	**Algebra and Calculus**	
1.1	Complex numbers	Include: • complex numbers expressed in the form $r(\cos\theta + i\sin\theta)$, $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$ • multiplication and division of two complex numbers expressed in polar form and their geometrical interpretations • loci of simple equations and inequalities such as $\lvert z-c \rvert \leqslant r$, $\lvert z-a \rvert = \lvert z-b \rvert$ and $\arg(z-a) = \alpha$ (excluding loci of $\lvert z-a \rvert = k\lvert z-b \rvert$, where $k \neq 1$ and $\arg(z-a) - \arg(z-b) = \alpha$) • use of de Moivre's theorem to find the powers and nth roots of a complex number, and to derive trigonometric identities
1.2	Polar coordinates	Include: • simple polar curves (for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals; and where r is non-negative throughout the domain) • use of formula $A = \dfrac{1}{2}\displaystyle\int_{\alpha}^{\beta} r^2 \, d\theta$ for the area of a sector • arc length of curves defined in polar form
1.3	Applications of definite integrals	Include: • arc length of curves defined in cartesian form • volume of revolution about the x- or y-axis for curves defined in cartesian form using discs or shells as appropriate • surface area of revolution about the x- or y-axis for curves defined in cartesian form
1.4	Functions of two variables	Include: • functions of two variables and surfaces $z = f(x,y)$ • first order and second order partial derivatives • minimum, maximum and saddle points • normal and tangent planes and local linearisation • directional derivatives and gradient • quadratic approximations • local maximum and minimum problems Exclude Jacobian matrix, Lagrange multipliers and constrained optimisation.

	Topic/Sub-topics	Content
1.5	Differential equations	Include: • analytical solution of first order and second order linear differential equations of the form: (i) $\dfrac{dy}{dx} + p(x)y = q(x)$, using an integrating factor (ii) $\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + by = 0$, where $a, b \in \mathbb{R}$ (iii) $\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + by = f(x)$, where $a, b \in \mathbb{R}$ and f(x) is a polynomial or pe^{kx} or $p\cos(kx) + q\sin(kx)$ including those that can be reduced to the above by means of a given substitution • relationship between the solution of a non-homogenous equation and the associated homogenous equation • family of solution curves • phase lines and slope fields* • exponential growth model • logistic growth model, equilibrium points and their stability, and harvesting
2	**Discrete Mathematics, Matrices and Numerical Methods**	
2.1	Recurrence relations	Include: • behaviour of a sequence, such as the limiting behaviour of a sequence • solution of (i) First order linear (homogeneous and non-homogeneous) recurrence relations with constant coefficients of the form $u_n = au_{n-1} + b, \quad a,b \in \mathbb{R}, \ a \neq 0$ (ii) Second order linear homogeneous recurrence relations with constant coefficients including those that can be transformed to the above by means of a given substitution • modelling with recurrence relations of the forms above

* For teaching and learning only.

	Topic/Sub-topics	Content
2.2	Matrices and linear spaces	Include: • use of matrices to represent a set of linear equations • operations on 3×3 matrices • determinant of a square matrix and inverse of a non-singular matrix (2×2 and 3×3 matrices only) • use of matrices to solve a set of linear equations (including row reduction and echelon forms, and geometrical interpretation of the solution) • linear transformations and matrices from $\mathbb{R}^n \rightarrow \mathbb{R}^m$, where $n, m \leqslant 3$ • eigenvalues and eigenvectors of square matrices (2×2 and 3×3 matrices, restricted to cases where the eigenvalues are real) • diagonalisation of a square matrix $\mathbf{M}$ by expressing the matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a diagonal matrix of eigenvalues and $\mathbf{Q}$ is a matrix whose columns are eigenvectors, and use of this expression such as to find the powers of $\mathbf{M}$ • linear spaces and subspaces, and the axioms (restricted to spaces of finite dimension over the field of real numbers only) • linear independence and span • basis and dimension (in simple cases), including use of terms such as 'column space', 'row space', 'range space' and 'null space' • rank of a square matrix and relation between rank, dimension of null space and order of the matrix
2.3	Numerical methods	Include: • location of roots of an equation by simple graphical or numerical methods • approximation of roots of equations using linear interpolation and Newton-Raphson method, *with the aid of a computer*, including cases where each method fails to converge to the required root • iterations involving recurrence relations of the form $x_{n+1} = \mathrm{F}(x_n)$, including cases where the method fails to converge • approximation of integral of a function using the trapezium rule and Simpson's rule, *with the aid of a computer* • approximation of solutions of first order differential equations using Euler method (including the use of the improved Euler formula), *with the aid of a computer*

	Topic/Sub-topics	Content
SECTION B: PROBABILITY AND STATISTICS		
3	**Probability and Statistics**	
3.1	Discrete random variables	Include: • use of Poisson distribution $\mathrm{Po}(\mu)$ and geometric distribution $\mathrm{Geo}(p)$ as probability models, including conditions under which each distribution is a suitable model • mean and variance for Poisson and geometric distributions • additive property of the Poisson distribution
3.2	Continuous random variables	Include: • probability density function of a continuous random variable and its mean and variance (includes 'piecewise' probability density function) • cumulative distribution function and its relationship with the probability density function • concepts of median and mode of a continuous random variable • use of the result $\mathrm{E}(\mathrm{g}(x)) = \int_{-\infty}^{\infty} \mathrm{g}(x)\mathrm{f}(x)\,\mathrm{d}x$ in simple cases, where $\mathrm{f}(x)$ is the probability density function of X and $\mathrm{g}(x)$ is a function of X • uniform distribution and exponential distribution as probability models • relationship between Poisson and exponential distributions

	Topic/Sub-topics	Content

3.3	Hypothesis testing and Confidence intervals	Include:
		 • formulation of hypotheses and testing for a population mean using a small sample drawn from a normal population of unknown variance using a *t*-test
		 • formulation of hypotheses for the difference of population means, and apply, as appropriate:
		   - a paired sample *t*-test
		   - a test using a normal distribution
		 • contingency tables and χ^2-tests of:
		   - goodness of fit
		   - independence
		 (excluding Yates' correction for continuity)
		 • connection between confidence interval and hypothesis test
		 • confidence interval for the population mean based on:
		   - a random sample from a normal population of known variance
		   - a small random sample drawn from a normal population of unknown variance
		   - a large random sample from any population
		 • confidence interval for population proportion (including concept of sample proportion) from a large random sample
		 • interpretation of confidence intervals and the results of a hypothesis test in the context of the problem
		
		 Exclude the use of the term 'Type I error', concept of Type II error and power of a test.

3.4	Non-parametric tests	Include:
		 • formulation of hypotheses and testing for:
		   - a population median using Sign test
		   - identical probability distributions for two sampled populations in a paired difference design using Wilcoxon matched-pair signed rank test
		 • advantages and disadvantages of non-parametric tests
		
		 Exclude treatment of tied ranks.

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Singapore–Cambridge Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$n(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union

$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\not>$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\not<$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$\displaystyle\sum_{i=1}^{n} a_i$ $a_1 + a_2 + ... + a_n$

$\sqrt[n]{a}$ the nth root of the real number a

$|a|$ the modulus of the real number a

$n!$ n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$

$\dbinom{n}{r}$ the binomial coefficient $\dfrac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$

              $\dfrac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
f(x)	the value of the function f at x
f: $A \rightarrow B$	f is a function under which each element of set A has an image in set B
f: $x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
gf	the composite function of f and g which is defined by gf(x) = g(f(x))
$\lim_{x \to a}$ f(x)	the limit of f(x) as x tends to a
Δx ; δx	an increment of x
$\dfrac{dy}{dx}$	the derivative of y with respect to x
$\dfrac{d^n y}{dx^n}$	the nth derivative of y with respect to x
f'(x), f''(x), …, f$^{(n)}$(x)	the first, second, … nth derivatives of f(x) with respect to x
$\displaystyle\int y\, dx$	indefinite integral of y with respect to x
$\displaystyle\int_a^b y\, dx$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \dots$	the first, second, …derivatives of x with respect to time
$\dfrac{\partial z}{\partial x}$	the partial derivative of z with respect to x
$\dfrac{\partial^2 z}{\partial y \partial x}$	the partial derivative of z with respect to x then with respect to y
f$_x$	the partial derivative of f with respect to x
f$_{xy}$	the partial derivative of f with respect to x then with respect to y

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
ln x	natural logarithm of x
lg x	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	} the circular functions
sin^{-1}, cos^{-1}, tan^{-1} cosec^{-1}, sec^{-1}, cot^{-1}	} the inverse circular functions

7. *Complex Numbers*

i	the square root of -1
z	a complex number, z $= x + iy$
	$= r(\cos\theta + i\sin\theta), r \in \mathbb{R}^+$
	$= re^{i\theta}, r \in \mathbb{R}^+$
Re z	the real part of z, Re $(x + iy) = x$
Im z	the imaginary part of z, Im $(x + iy) = y$
$\|z\|$	the modulus of z, $\|x+iy\| = \sqrt{x^2 + y^2}$, $\|r(\cos\theta + i\sin\theta)\| = r$
arg z	the argument of z, $\arg(r(\cos\theta + i\sin\theta)) = \theta, -\pi < \theta \leqslant \pi$
$z*$	the complex conjugate of z, $(x + iy)* = x - iy$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\begin{pmatrix} x \\ y \end{pmatrix}$	a column vector in xy-plane
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	a column vector in xyz-space
$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\mathbf{\hat{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i, j, k}$	unit vectors in the directions of the cartesian coordinate axes
$\|\mathbf{a}\|$	the magnitude of $\mathbf{a}$
$\|\overrightarrow{AB}\|$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a \times b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
P(A)	probability of the event A
A'	complement of the event A, the event 'not A'
P($A \mid B$)	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
x_1, x_2, …	observations
f_1, f_2,…	frequencies with which the observations, x_1, x_2, …occur
p(x)	the value of the probability function P($X = x$) of the discrete random variable X
p_1, p_2,…	probabilities of the values x_1, x_2, …of the discrete random variable X
f(x), g(x)…	the value of the probability density function of the continuous random variable X
F(x), G(x)…	the value of the (cumulative) distribution function P($X \leqslant x$) of the random variable X
E(X)	expectation of the random variable X
E[g(X)]	expectation of g(X)
Var(X)	variance of the random variable X
B(n, p)	binomial distribution, parameters n and p
Po(μ)	Poisson distribution, mean μ
Geo(p)	Geometric distribution, mean $\dfrac{1}{p}$
N(μ, σ^2)	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\overline{x}$	sample mean
s^2	unbiased estimate of population variance from a sample
r	linear product-moment correlation coefficient for a sample